# 3.14 Dot Product of Vectors

Definition: Dot Product

The **dot product** of **u** = $\langle u_1, u_2 \rangle$ and **v** = $\langle v_1, v_2 \rangle$ is

**u•v** = $u_1v_1 + u_2v_2$ .

**Properties of the Dot Product**

Let **u**, **v**, and **w** be vectors and let c be a scalar.

1. **u • v = v • u**

2. **u • u =** $|\mathbf{u}|^2$

3. **0 • u = 0**

4. **u • (v+w) = u • v + u • w**

   **(u+v) • w = u • w + v • w**

5. (c **u) • v = u • (c****v) = c(u • v)**

**Example:**

Find each dot product.

a) $\langle 3,4 \rangle \bullet \langle 5,2 \rangle$ = (3)(5) + (4)(2) = 23

b) $(2i - j) \bullet (3i - 5j)$ = (2)(3) + (-1)(-5) = 11

Example:

Using the dot product to find the length of a vector:

Find length of the vector **u =** $\langle 4,-3\rangle$**.**

$|u| = \sqrt{u \bullet u}$ so, $|\langle 4,-3\rangle| = \sqrt{\langle 4,-3\rangle \bullet \langle 4,-3\rangle} = \sqrt{(4)(4)+(-3)(-3)} = \sqrt{25} = 5$

## Angle Between Vectors

If $\theta$ is the angle between the nonzero vectors **u** and **v**, then

$$\cos\theta = \frac{u \bullet v}{|u||v|}$$

$$and \qquad \theta = \cos^{-1}\left(\frac{u \bullet v}{|u||v|}\right)$$

**Example:**

Find the angle between the vectors **u** and **v**.

**u** = $\langle 2,3\rangle$, **v** = $\langle -2,5\rangle$

Using the theorem above, $\cos\theta = \frac{u \bullet v}{|u||v|} = \frac{\langle 2,3\rangle \bullet \langle -2,5\rangle}{|\langle 2,3\rangle||\langle -2,5\rangle|} = \frac{11}{\sqrt{13}\sqrt{29}}$

so, $\theta = \cos^{-1}\left(\frac{11}{\sqrt{13}\sqrt{29}}\right) \approx 55.5°$

**Orthogonal Vectors (angle between vectors is 90°)**

The vectors **u** and **v** are **orthogonal** if and only if **u •v** = 0.

**Proving vectors are orthogonal:**

Example:

Prove that the vectors **u** = $\langle 2,3 \rangle$ and **v =** $\langle -6,4 \rangle$ are orthogonal.

We must show that the dot product is zero.

**u • v =** $\langle 2,3 \rangle \bullet \langle -6,4 \rangle = -12+12=0$**,**

therefore, the vectors are orthogonal.

*Vectors are **parallel** if **u** = *k***v** for some constant *k.*

If **v** = $\left\langle 3, 4 \right\rangle$ and **u** = $\left\langle 6, 8 \right\rangle$, then **u** and **v** are parallel because **u** = 2**v**.